An Electromechanical Model for PZT utilized in Energy Harvesting System

Komsak Komoljindakul, Siriroj Sirisukprasert
Department of Electrical Engineering
Kasetsart University
50 Ngamwongwan Rd., Chatuchak, Bangkok, 10900, Thailand
g5314501384@ku.ac.th, fengsir@ku.ac.th

Natchpong Hatti, Kanokvate Tungpimolrut
National Electronics and Computer Technology Center
National Science and Technology Development Agency
112, Klong 1, Klong Luang, Pathumthani, 12120, Thailand
natchpong.hatti@nectec.or.th,
kanokvate.tungpimolrut@nectec.or.th

Kesorn Pechrach
Ronsek Ltd.
7 Boundary Rd., Bishops Stortford, Herts, CM23, 5LE
United Kingdom
kesorn@ronsek.com

Poramate Manoonpong
Bernstein Center for Computational Neuroscience (BCCN)
Göttingen University
Friedruch-Hund Platz 1, Göttingen, 37077, Germany
poramate@manoonpong.com

***Abstract*— This paper proposes a new electromechanical model for piezoelectric ceramic lead zirconate titanate (PZT) employed in energy harvesting systems. The proposed model consists of a parallel resonant circuit, loss components and discrete switches to emulate behaviors of electrical energy generated from the PZT vibration. To verify the accuracy of the proposed model, its electrical behavior is experimentally compared with that of the prototype. The results indicate a significant consistency. Additionally, this paper demonstrates how the proposed model performs in energy harvesting system design process. The results confirmed that the proposed model can be practically utilized.**

Keywords- Piezoelectric Materials, Energy Harvesting Circuits, Parallel Resonant Circuits, Buck Converter

I. INTRODUCTION

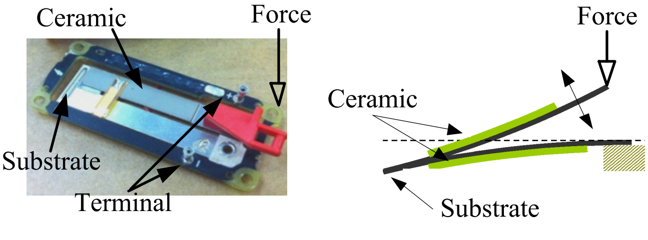
Currently, there are active research and development in utilizing alternative energy sources such as wind, solar, geothermal energy, etc. Nonetheless, for applications those require low power have a concept of using surplus vibration energy from, for example, running industrial electric motor or human motions [1]. To convert these mechanical energy to electrical energy, a well-known device so-called "piezoelectric" is employed. Electrical energy from the piezoelectric is, however, still very low; therefore, piezoelectric devices need to be integrated with energy harvesting circuits to achieve effectively officiate energy storage and energy transmission. The mechanical and electrical behaviors and characteristics of the piezoelectric devices have been studied. Its energy flow and impedance modeling have been investigated [2]. Due to its performance, PZT is considered as one of the promising piezoelectric. Commonly, the PZT devices are based on cantilever beam structure.

In this paper, the behaviors and characteristics of a novel structural piezoelectric ceramic lead zirconate titanate (PZT) shown in Fig. 1(a) will be studied. It comprises two piezoelectric ceramic plates (arms) connected together with a substrate as shown in Fig. 1(b). This structure results in higher energy conversion efficiency over a broad range of low frequency applications [3]. This paper proposes analysis and modeling techniques to emulate the electrical behaviors for the PZT based on its experimental results. The main parameters will be determined and modeled. To confirm its accuracy, the electrical behaviors of the proposed model is simulated by using PSPICE program and experimentally verified with the prototype. Moreover, to show the utilization of the proposed model, a buck-converter based harvesting system behavior is simulated as a test case.

II. ANALYSIS AND MODELING

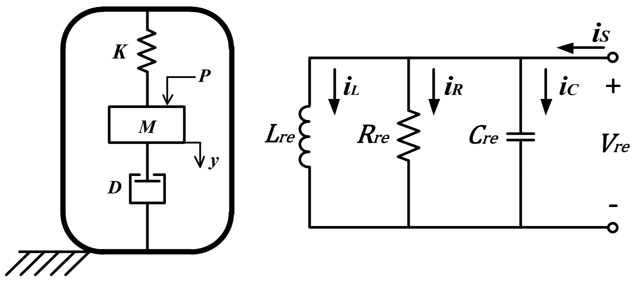
In previous research, a prototype for PZT with cantilever beam was modeled based on a dynamic analysis of the mechanical part and its response in voltage waves in an open-circuit test. It was also tested using load resistance to find its maximum power. The result revealed that an equivalent circuit inside the PZT was a series RLC circuit, normally found in general research articles.

In this paper, we aim to invent a new PZT model capable of emulating the same or better electrical behaviors in low frequency range operation.



(a) (b)

Figure 1. (a) Components and (b) a simple structure of the PZT.



(a) (b)

Figure 2. (a) Component of general mechanical system
(b) corresponding parallel circuit of an electrical system.

A. Mechanical Behavior of PZT

In order to find parameter which can reveal characteristics of PZT, it is important to understand its electrical energy generation resulted from mechanical energy by the use of vibration.

The modeling starts with the analogy of the mechanical and the electrical systems. Fig. 2(a) illustrates a basic mechanical system consists of applying force (P), a mass (M), a damper (D), a spring (K), and displacement (*y*) [4]. Theoretically, the response of this mechanical system can be used to represent the natural respond of the piezoelectric devices and is expressed as (1). To emulate the mechanical dynamic response of the proposed PZT in an electrical system, a parallel resonance circuit shown in Fig. 2(b) can be employed.

$$M\frac{dy^2}{dt} + D\frac{dy}{dt} + Ky = P \tag{1}$$

$$C\frac{d\varphi^2}{dt} + \frac{1}{R}\frac{d\varphi}{dt} + \frac{1}{L}\varphi = i_S \tag{2}$$

The mechanical components in (1) analogies to electrical component as shown in Table I.

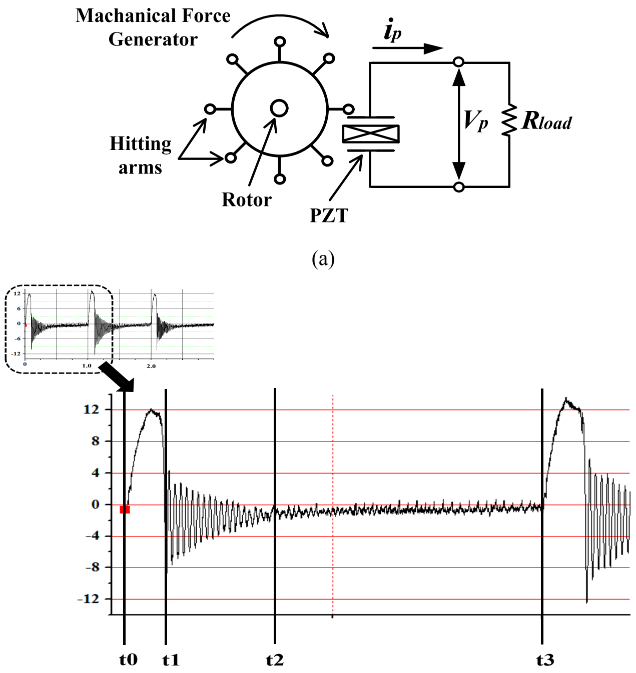
TABLE I. COMPARE PARAMETER FORM EQU. (1) AND (2)

Mechanical Part	Electrical Part
Force: P	Current: is
Mass: M	Capacitance: C
Displacement: y	Electric field: φ
Velocity: $\dot{y}$	Voltage: Vre
Damper: D	Reciprocal of Resistance: $1/R$
Spring Constant: K	Reciprocal of Inductance: $1/L$

B. Electrical Model Development

This paper presents the electrical modeling technique for the new PZT (Lead Zirconate Titanate). Firstly, the electro-mechanical characteristic of the studied PZT will be analyzed based on experiments. To investigate its energy conversion behavior, an experiment shown in Fig. 3(a) has been setup. A DC motor is employed as the mechanical force generator, which is controlled to hit the PZT at frequency of 1 Hz. The output voltage waveform, V_P, generated from the PZT is measured. Fig. 3(b) is the result voltage waveform from the experiment. The result revealed that there were two response intervals in the generated voltage waveforms caused by,

- t0-t1: hitting force causing impact on the PZT and,
- t1-t2: natural vibration of the PZT causing decay voltage oscillation with resonance frequency, which depends on the piezoelectric materials and structure.



(a)

(b)

Figure 3. (a) Experimental setup with hitting force
(b) open-circuit voltage waveform of the PZT prototype.

Next, the response of the PZT will be emulated by the following techniques. In general, the electrical energy generated by PZT can be either in the form of voltage source or current source. The voltage source is chosen in the proposed model. The main internal parameters, i.e. the output resistance R_P and the output capacitance C_P, are measured by a RCL meter (Fluke PM6306).

Based on Fig. 3(b), the second duration of the voltage response can be modeled as a second-order parallel RLC circuit. Its resonance equation can be express as a function of complex frequency (S_1 and S_2), which is later used to determine the values of *R*, *L* and *C*. Furthermore, the voltage response in this duration is found to be underdamped. Theoretically, the underdamped voltage response occurs when the damping factor, ξ is less than one. The damping factor and the roots S_1 and S_2 can be expressed as

$$\xi = \frac{\alpha^2}{\omega_0^2} = \frac{1}{2R}\sqrt{\frac{L}{C}} \tag{3}$$

$$s_{1,2} = -\alpha \pm j\omega_d \tag{4}$$

,where

$$\alpha = \frac{1}{2 R_{re} C_{re}} \tag{5}$$

And the damped radian frequency is

$$\omega_d = \sqrt{(\omega_0^2 - \alpha^2)} \tag{6}$$

where the resonant radian frequency is

$$\omega_0 = \frac{1}{\sqrt{L_{re} C_{re}}} \tag{7}$$

Based on the above analysis and the experimental results, the proposed model of studied PZT system is illustrated in Fig.4. The mechanical energy loss is modeled as series resistor R_{in1}. The coupling energy path from the input terminal to the output terminal of PZT is formed by the resistor R_{in2} and the capacitor C_{in}. The duration of the impact caused by the hitting force and the oscillation period can be controlled by SW1 and SW2. These two switches are complementary. SW1 closes when the PZT arm is hit and opens when the PZT is releasing the energy. SW2 is closed at this moment to emulate the resonated energy. A pulse voltage source, *Vpulse* is the representative of applied mechanical force, whose frequency depends on how often the PZT hit by the mechanical force generator.

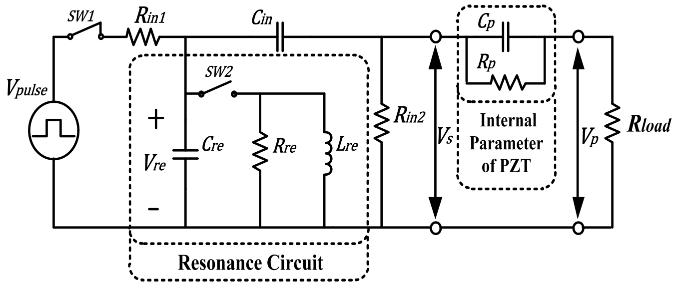


Figure 4. Electromechanical model of the PZT.

III. SIMULATION AND EXPERIMENT RESULTS

A. *Verification of the Proposed PZT System Model*

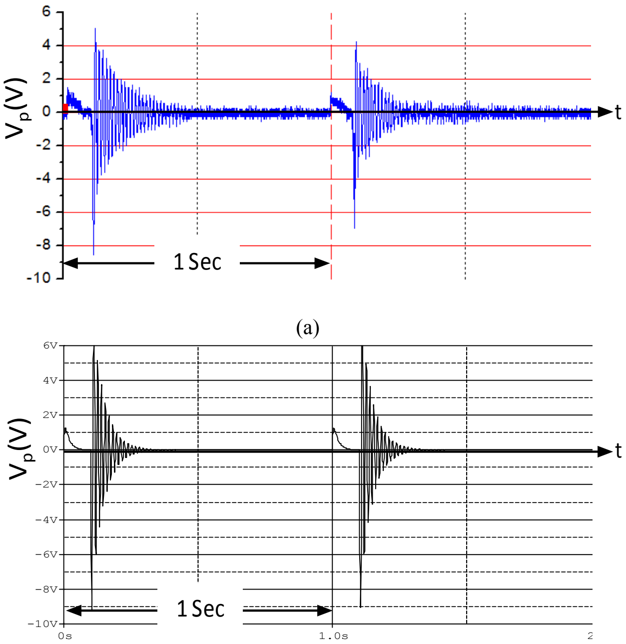
To verify the accuracy of the proposed model, PSPICE is employed as the simulation tool. The simulation and the experimental results of the PZT system will then be compared. Appling equations (3)-(7), all related parameters are calculated and its values are shown in Table II. The value of inductor *Lre* and capacitor *Cre* are determined by the resonant frequency of the PZT and (7). In this case, the PZT system is connected with five different resistive loads as shown in Table II. The case of resistive load of 50kΩ is used in the following explanation.

Fig. 5(a) and (b) shows the experimental and simulation results, respectively. The behavior of the PZT from the simulation and the experimental results are significantly similar, except no noise components in the simulation.

To further verify, the experimental results shown in Fig.5(a) is investigated by using Fast Fourier Transform (FFT) to identify its frequency components. The result is illustrated in Fig. 6(a). Likewise, Fig. 6(b) shows the frequency spectrum of the simulation result, which is extracted from FFT in PSPICE program. From the results, the resonant frequencies of the experiment and the simulation are at 75Hz and 71Hz, respectively. This leads to the error of approximately 5.34%.

TABLE II. PARAMETERS AND THEIR VALUES USED FOR VERIFICATION.

Parameters	Values
Resistive Load (kΩ)	20, 50, 100, 200, 300
Applied force frequency (Hz)	0.5, 2
Resistance for coupling energy: *Rin1*	5 Ω
Resistance for coupling energy: *Rin2*	500 kΩ
Resistance for resonance: *Rre*	6 Ω
Internal resistance of the PZT: *Rp*	2 MΩ
Inductance for resonance: *Lre*	1305 μH
Capacitance for coupling energy: *Cin*	0.2 μF
Capacitance for resonance: *Cre*	3450 μF
Internal capacitance of the PZT: *Cp*	48.2 nF
Resonance frequency: f_0	75 Hz
Damping factor: ξ	0.059



(a)

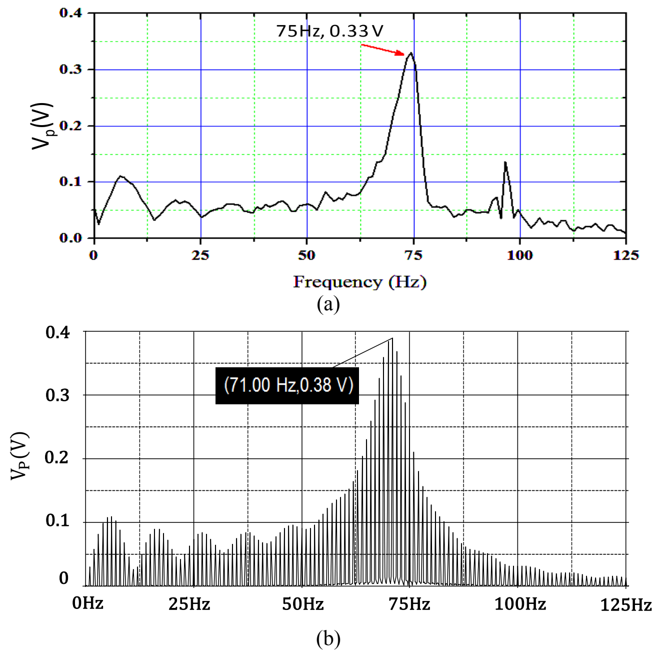
(b)

Figure 5. Voltage waveform of the PZT
(a) experiment reults (b) simulation reults from PSPICE.

Table III shows the percentage error of the generated electrical energy from the proposed model compared with those from the PZT prototype. In this particular comparision, the maximum and minimum errors are 15.68% and 0.26%, respectively.

TABLE III. PERCENTAGE ERROR OF ELECTRICAL ENERGY DELIVERY FROM THE PZT.

R(kΩ) / F(Hz)	20	50	100	200	300
0.5	10.02	**15.68**	-4.28	-2.03	**-0.26**
2.0	-13.52	-5.24	-12.21	-7.23	-7.04



(a)

(b)

Figure 6. Frequency spectrum of PZT voltage (a) experiment results (b) simulation results.

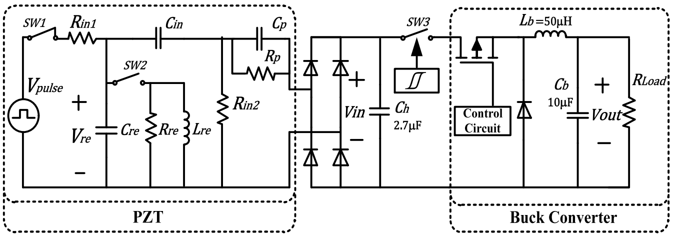


Figure 7. The simulated energy harvesting system with the proposed model.

B. Energy Harvesing System with Buck converter

After verify its accuracy, the proposed model of PZT system is utilized to simulate the electrical behavior of an energy harvesting system as shown in Fig. 7. In this case, a feedback-controlled buck converter is used as the output voltage regulator. The voltage generated by the PZT is rectified by a full-wave diode circuit, which is normally used in energy harvesting applications. The un-regulated DC voltage is then fed as the input of buck converter.

The simulation results illustrate in Fig. 8. At *t0*, the mechanical force applied to the PZT with no electrical load. The electrical energy generated by the PZT then starts accumulating in the capacitor, C_h. Right after the capacitor voltage reaching 3.3V at *t1*, switch SW3 is closed, and the buck converter starts operating and regulates the output voltage at 3.0V. The operation of the SW3 is controlled based on hysteresis loop. In case of 2.5Hz applying force, the steady-state input voltage V_{in} is designed to be 9.0V. At t_2, a 900kΩ resistive load is connected to the buck converter. The power delivered to resistive load is 10µW as shown in Fig. 8. The applied mechanical force is stopped at *t3*. As a result, the electrical energy stored in C_h starts dissipating to the load causing V_{in} deceased. When V_{in} is lower than 3.3V, the SW3 is turned off and the buck converter stops operating. Voltage V_{out} is then not regulated and decays to zero. The simulation results indicate that the proposed model can be practically utilized in the design process of an energy harvesting system.

IV. CONCLUSIONS

This paper proposed a new electromechanical modeling technique for PZT, which is employed in energy harvesting systems. The model was derived from a prototype's mechanical-to-electrical energy conversion characteristics. The accuracy of the proposed model has been experimentally verified. The comparison results have indicated that the proposed model is consistent with the electrical characteristic of the studied PZT. This paper also demonstrated how the proposed model performs with the buck converter. The simulation results confirmed that the proposed model can be practically utilized in the energy harvesting system design process. Errors of the proposed model, however, exist. Further investigation and improvement are needed.

REFERENCES

[1] K. Pechrach, P. Manoonpong, F. Woegoetter, K. Tungpimolrut, N. Hatti, J. Phontip and K. Komoljindakul, "Piezoelectric Energy Harvesting for Self Power Generation of Upper and Lower Prosthetic Legs," Piezo2011, Sestriere, Italy, February - March 2011.

[2] J. R. Liang and W. H. Liao, "Energy flow in piezoelectric energy harvesting systems," J. Smart Mater. Struct., vol.20, no.1, act. no. 105005 (11 pp.), January 2011.

[3] K. Pechrach, J.W. McBride and P.M. Weaver, "Arc root mobility on piezo actuated contacts in miniature circuit breakers," IEEE Holm Conference on Electrical Contacts, no. 8-10, pp. 76-83, September 2003.

[4] N. Hatti, K. Tungpimolrut, J. Phontip, K. Pechrach, P. Manoonpong and K. Komoljindakul, "A PZT Modeling for Energy Harvesting Circuit," IEEE International Symposium on Applications of Ferroelectrics, ISAF-PFM, vol.20, Vancouver, Canada, July 2011.

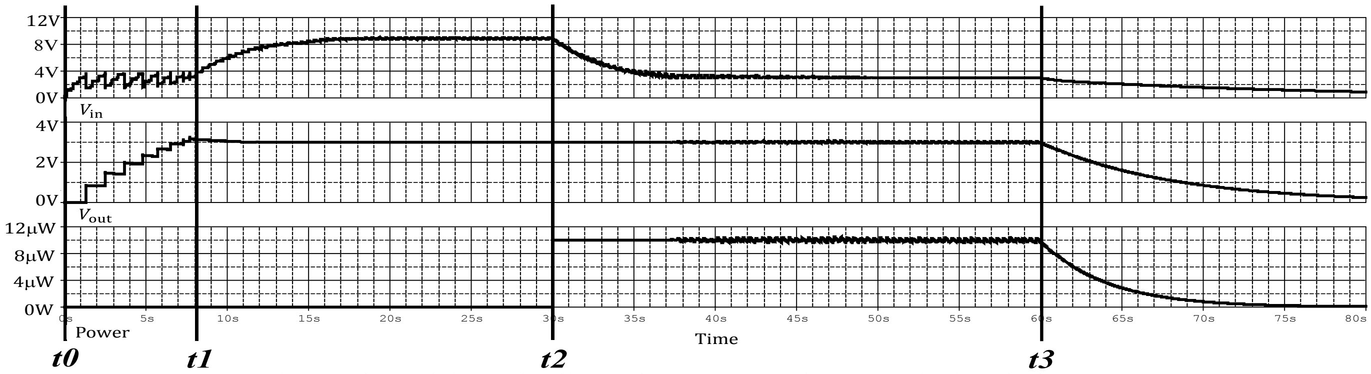


Figure 8. The simulation results of input voltage V_{in}, output voltage, V_{out} and resistive load power.